# Averaging Attacks on Bounded Perturbation Algorithms

Hassan Jameel Asghar & Dali Kaafar

Macquarie University, Australia {hassan.asghar, dali.kaafar}@mq.edu.au

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#### Abstract

We describe and evaluate an attack that reconstructs the histogram of any target attribute of a sensitive dataset which can only be queried through a type of privacy-preserving algorithms which we call bounded perturbation algorithms. A defining property of such an algorithm is that it perturbs answers to the queries by adding noise distributed within a bounded range (possibly undisclosed). Other key properties of the algorithm include only allowing restricted queries (enforced via an interface), suppressing answers to queries which are only satisfied by a small group of individuals (e.g., by returning a zero as an answer), and adding the same perturbation to two queries which are satisfied by the same set of individuals (to thwart differencing or averaging attacks). We assume an attacker (say, a curious analyst) who is given oracle access to the algorithm via an interface. The attacker may not know the perturbation parameter r (specifying the noise range). We describe two attacks on the algorithm. Both attacks are based on carefully constructing (different) queries that evaluate to the same answer. The first attack finds the hidden perturbation parameter (if it is assumed not to be public knowledge). The second attack removes the noise to obtain the original answer of some (counting) query of choice. We also show how to use this attack to find the number of individuals in the dataset with a target attribute value a of any attribute A, and then for all attribute values  $a_i \in A$ . None of the attacks presented here depend on any background information.

As a practical illustration of the attack, we evaluate it by querying a synthetic dataset via an API to the bounded perturbation algorithm [15] used in the Australian Bureau of Statistics (ABS) TableBuilder tool. While the attack is also applicable to the actual Australian census data available though TableBuilder, for ethical considerations we only show the success of the attack on synthetic data. We note, however, that at the time of this writing the perturbation method used in the online ABS TableBuilder tool is vulnerable to this attack.

Our results show that a small value of the perturbation parameter (desirable from a utility point of view), e.g., perturbing answers by uniformly sampling (integral) noise within range  $\leq \pm 5$ , can be retrieved with less than 200 queries with a probability of more than 0.95. This probability reaches 1 exponentially with only a linear increase in the number of queries. Furthermore, we show that the true count behind any target attribute value can be retrieved with only 400 queries with a probability of more than 0.96, and the entire column of more than 100 different attribute values can be retrieved with a corresponding linear increase in the number of queries. While we discuss possible ways to patch bounded perturbation algorithms to defend against our attacks, we stress that the algorithm might still be susceptible to other similar attacks. A better approach would be to carefully upscale noise as a function of the number of queries allowed to control privacy leakage. Our attacks are a practical illustration of the (informal) fundamental law of information recovery which states that "overly accurate estimates of too many statistics completely destroys privacy" [2, 6].

## 1 Introduction

We consider privacy-preserving algorithms that return *noisy* answers to queries on sensitive data, where the noise is strictly bounded between an interval parameterised by a *perturbation parameter*. Our focus

<sup>&</sup>lt;sup>1</sup>TableBuilder is an online tool which enables users to create tables, graphs and maps of Australian census data. See http://www.abs.gov.au/websitedbs/censushome.nsf/home/tablebuilder.

is restricted to algorithms that (privately) answer counting queries. An example counting query is: "How many people in the dataset are aged 25 and live in the suburb of Redfern in New South Wales?" An example of such privacy-preserving algorithms is the perturbation algorithm employed by the TableBuilder tool from the Australian Bureau of Statistics (ABS), which allows access to the Australian population census data. We shall call this algorithm the TBE algorithm named after its authors [15]. The TBE algorithm and similar bounded perturbation algorithms are built on certain principles to address privacy and utility concerns, outlined below.

- Access to sensitive data is only allowed through a *restricted* query interface. This limits the types of queries that can be executed via the underlying (privacy-preserving) algorithm, therefore minimising information leakage by ensuring that the (effective) query language is not rich enough. A rich query language would require query auditing to ensure privacy; such auditing may not even be programmable [5].
- The noise added to the queries is bounded within a predetermined range, say ±3 of the actual answer. From a privacy angle this adds uncertainty if the (adversarial) analyst is trying to run a query on certain attributes in the dataset to determine characteristics of its target individual. From a utility point of view, the bounded noise ensures that the noise never overwhelms the true statistics.
- The algorithm suppresses low non-zero counts (e.g., by returning 0). This makes it hard for an analyst to know if certain characteristics (combination of attributes in the dataset) are shown by its target individual(s) or not.
- The algorithm adds the exact same noise if the answers returned by two queries are contributed by the same set of *contributors*. A contributor to a query on a set of attributes is any individual that satisfies the query. This is a defence against differencing attacks [8], where the analyst cannot pose two different queries with the same contributors to minimise the noise by averaging (if different).

Contributions. We show an attack that retrieves the entire histogram of a target attribute from a dataset which can only be queried through the above mentioned privacy-preserving algorithm. Our attack relies on carefully constructing queries that yield the same (true) answer and averaging them over all queries to eliminate noise. Furthermore, in cases where it may be argued that the perturbation parameter is not public information, we show an attack that retrieves the exact (hidden) perturbation parameter. We note that there is a specific class of attacks known as reconstruction attacks that seeks to reconstruct a whole (target) column of a sensitive dataset [7, 2, 4, 11]. Algorithms that allow overly accurate answers to too many linear queries are susceptible to these attacks. More precisely, algorithms who return noisy answers with noise bounded within  $o(\sqrt{n})$ , where n is the number of rows (individuals) in the dataset, succumb to these attacks. These attacks may or may not be applicable to the above mentioned class of algorithms (due to same amount of noise added to the same set of contributors and lower counts being fixed to zero) and may require a much larger set of queries. Nevertheless, our attack can be considered as an instance of a reconstruction attack tailored to the specific class of algorithms described above. The attacks presented do not depend on any background knowledge of the dataset, i.e., they are dataset independent, and hence applicable to any underlying dataset.

**Results.** For both attacks, i.e., finding the hidden perturbation and removing noise, we derive exact expressions for the success probabilities as a function of the perturbation parameter and the number of queries to the algorithm. We also evaluate the noise removing attack on a synthetic dataset queried through an API to the TBE algorithm. Our results (both theoretical and experimental) show that any perturbation parameter less than or equal to 10 can be retrieved with probability  $\approx 0.90$  with only up to 1,000 queries.

<sup>&</sup>lt;sup>2</sup>See http://www.abs.gov.au/websitedbs/censushome.nsf/home/tablebuilder. At the time of this writing, there are two flavours of TableBuilder. The first is TableBuilder Basic & Pro, which requires registration. After the registration request is approved, the user can login to use the TableBuilder tool. The second flavour is for guests, called TableBuilder Guest. This can be accessed by users without registration and provides access to fewer data items. The attacks mentioned in this paper are applicable on both flavours.

<sup>&</sup>lt;sup>3</sup>Barring a few mild assumptions on the domain of the dataset, e.g., the existence of an attribute with more than 2 attribute values (Section 4.1).

Furthermore, we are able to recover a smaller perturbation parameter (5), which is desirable for utility, with only 200 queries with a probability of more than 0.95. Using the same API, we retrieve an entire histogram of a target column of the synthetic dataset with more than 107 attribute values through only 400 queries per attribute value (via the noise removing attack). The attack also successfully retrieves suppressed counts (low counts returned as 0), and hence distinguishes between actual zeros and suppressed zeros.

Application to the ABS TableBuilder. Our use of the API to query the TBE algorithm simulates the setting of the ABS TableBuilder tool providing access to the Australian census data. The TableBuilder tool does not currently have a programmable API, and can only be accessed via a web interface. Naturally, our attack does not need the use of an API to be successful though. The attack in practice can still be launched by either manually querying TableBuilder to construct tables or more realistically, by crafting web queries from scripts to directly query the JavaScript programs behind the web interface. We chose to use the simulated setting for a quicker illustration of the attack and more importantly due to ethical considerations; the census data being highly sensitive.

## 2 Preliminaries

We model the database D as a set of rows of data, each belonging to a unique individual from a finite set of individuals U. Thus, the size of the dataset is the same as the size of the set U, i.e., |D| = |U|. The data from an individual  $u \in U$  is represented as a row  $x \in D$ . We denote the link by x = data(u). The row x is a member of some domain  $\mathbb{D}$ .

**Definition 1** (Query). A query  $q: \mathbb{D} \to \{0,1\}$  is defined as a predicate on the rows  $x \in D$ . The query's result on the dataset D is defined as

$$q(D) = \sum_{x \in D} q(x).$$

For any two queries  $q_1$  and  $q_2$ , we denote by  $q_1 \wedge q_2$  the predicate that evaluates to 1 on a row if and only only if both  $q_1$  and  $q_2$  evaluate to 1 on the row. Likewise we denote by  $q_1 \vee q_2$  the predicate that evaluates to 1 if either  $q_1$  or  $q_2$  or both evaluate to 1.

We will often omit the argument of q, i.e., D, since we are concerned with a single dataset in this document.

**Definition 2** (Contributors). A contributor of a query q is any individual  $u \in U$  such that q(x) = 1, where x = data(u). The set of contributors of a query q, denoted C(q) is defined as

$$C(q) = \{u \in U : q(x) = 1, \text{ where } x = \text{data}(u)\}\$$

Two queries  $q_1$  and  $q_2$  are said to have the same contributors if  $C(q_1) = C(q_2)$ . Otherwise they are said to have different contributors.

Note that having different contributors does not mean that  $C(q_1) \cap C(q_2) = \emptyset$ . Also note that two different queries (different predicates)  $q_1$  and  $q_2$  may also have the same contributors. The dataset D is vertically divided into attributes. Let A denote one such attribute, and let |A| denote its cardinality, i.e., the number of attribute values of A. Let  $a \in A$  be an attribute value. We assume that the data of each  $u \in U$  takes on only one value from A. The query  $q_a$  is defined as the predicate which evaluates to 1 if the row has value a under A. Let  $A' \subseteq A$ , then  $q_{A'}$  is defined as

$$q_{A'} = \bigvee_{a \in A'} (q_a)$$

We also denote the trivial query  $q_{\emptyset}$ , which evaluates to 1 on every row. Hence  $q_{\emptyset}(D) = n$ . Also, note that  $q_A(D) = n$  for every attribute A of D. Clearly, in both cases the set of contributors is the entire user set U.

**Example 1.** Table 1 shows a dataset D with 6 users. Queries  $q_{\text{Redfern}}$  and  $q_{20\text{-}29}$  both evaluate to 3. Also note that  $C(q_{\text{Redfern}}) = C(q_{20\text{-}29}) = \{1, 2, 4\}$ , and thus the two queries have the same set of contributors. On the other hand,  $C(q_{\text{Redfern}}) \neq C(q_{\text{M}})$ . We have  $(q_{\text{Redfern}} \land q_{\text{M}})(D) = 2$ , and  $(q_{\text{Redfern}} \lor q_{\text{M}})(D) = 4$ . Let  $A' = \{\text{Redfern}, \text{Newtown}\}$ . Then  $q_{A'}(D) = 4$ .

<sup>&</sup>lt;sup>4</sup>This is in fact the definition of a counting query. The queries in this document are restricted to counting queries.

Table 1: An example	database $D$ with	U	=6 users.
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U	Suburb	Age	Gender
1	Redfern	20-29	M
2	Redfern	20-29	M
3	Newtown	30-39	F
4	Redfern	20-29	F
5	Surry Hills	40-49	M
6	Darlinghurst	70-79	F

## 3 Privacy Algorithms

Our focus is on a particular privacy algorithm (mechanism) that returns (perturbed) answers to queries q on the database D, where the queries are as defined in Definition 1. We call the algorithm the bounded noisy counts algorithm. The algorithm returns the answer to a query q by adding bounded noise e from the uniform distribution over the set of integers in the interval [-r, r], where r is some positive integer. We shall denote the set of integers in [-r, r] by  $\mathbb{Z}_{\pm r}$  and the (discrete) uniform distribution over  $\mathbb{Z}_{\pm r}$  by  $\mathbb{U}_{\pm r}$ . The algorithm also has two exceptional cases:

- 1. If the answer to q is less than a parameter  $s \geq r$ , then the answer returned is exactly 0.
- 2. If two queries  $q_1$  and  $q_2$  have the same contributors, then the noise e added to the two queries is the same.

The algorithm therefore is a stateful algorithm where the state consists of a dictionary of subsets of contributors and the corresponding noise. We denote this algorithm by  $\mathcal{M}_{r,s}$  and on any input query q denote its output as  $\mathcal{M}_{r,s}(q)$ . The algorithm is described in Algorithm 1.

```
Input: The query q, perturbation parameter r, suppression parameter s \ge r.
   State: A noise dictionary, denoted nd, with keys from subsets of U and values in \mathbb{Z}_{\pm r}.
 1 Evaluate C(q) and let n = q(D).
 2 if n \leq s then
       return 0.
 3
 4 else
        if C(q) \in \mathsf{nd} \ \mathsf{then}
 5
            obtain noise e \leftarrow \mathsf{nd}(C(q)).
 6
 7
            return n + e.
 8
        else
            sample e \sim \mathbb{U}_{\pm r}.
 9
            add entry \operatorname{nd}(C(q)) = e.
10
            return n + e.
11
```

**Algorithm 1:** The Bounded Noisy Counts Algorithm  $\mathcal{M}_{r,s}$ .

From now on, we shall drop the subscripts r and s, and denote the algorithm simply as  $\mathcal{M}$ . Following properties are direct consequences of the algorithm.

**Proposition 1.** Let  $\alpha \leftarrow \mathcal{M}(q)$  be the answer returned by  $\mathcal{M}$  on some query q. Let n = q(D). Then

(a)  $\alpha \geq 0$ , (b)  $n - s \leq \alpha \leq n + r$ . (c) If  $\alpha > 0$  then  $C(q) \neq \emptyset$ .

*Proof.* Let e be the noise added by  $\mathcal{M}$ . (a) First assume n > s. Then since  $e \in [-r, r]$ ,  $\alpha = n + e \ge n - r \ge n - s > 0$ . Here we have used the fact that  $s \ge r$ . If  $n \le s$ , then  $\alpha = 0$  (step 2 of the algorithm). (b) Again,

first assume n > s. Then  $n - r \le \alpha \le n + r \Rightarrow n - s < \alpha \le n + r$ . Now, if  $n \le s$ , then  $\alpha = 0$ . Trivially,  $n - s \le 0 = \alpha$ . (c) If C(q) is empty, then |C(q)| = 0 < s, and hence  $\mathcal{M}$  should always return a 0 in this case.

The above algorithm is used as a subroutine by another algorithm which we call the *attribute analyser*. This algorithm helps the querier analyse multiple attribute values under an attribute at once. Let **B** denote a tuple of (one or more) attributes in D. Let  $\mathbf{b} \in \mathbf{B}$  denote the a vector of attribute values, where the *i*th value (element) corresponds to an attribute value from the *i*th attribute in **B**. Let  $q_{\mathbf{b}}$  denote the predicate that evaluates to 1 if and only if the row satisfies all values in  $\mathbf{b}$ . This algorithm takes an attribute value vector  $\mathbf{b} \in \mathbf{B}$ , and a subset of attribute values  $A' \subseteq A$ , where  $A \notin \mathbf{B}$ . Let |A'| = m. The algorithm then runs  $\mathcal{M}$  on (i) each of the queries  $q_{\mathbf{b}} \wedge q_{a_i}$  where  $a_i \in A', i \in \{1, \ldots, m\}$  obtaining answers  $\alpha_i$ , and (ii) on the *total* query  $q_{\mathbf{b}} \wedge q_{A'}$ , obtaining the answer  $\alpha_{A'}$ . It then returns the answer vector  $(\alpha_1, \ldots, \alpha_m, \alpha_{A'})$ .

**Input:** Attribute value vector  $\mathbf{b} \in \mathbf{B}$ , attribute subset  $A' \subseteq A$  of cardinality m (where  $A \notin \mathbf{B}$ ).

```
1 for i=1 to m do

2 | Let a_i be the ith element in A'.

3 | Obtain \alpha_i \leftarrow \mathcal{M}(q_{\mathbf{b}} \wedge q_{a_i}).

4 Obtain \alpha_{A'} \leftarrow \mathcal{M}(q_{\mathbf{b}} \wedge q_{A'}).

5 return (\alpha_1, \ldots, \alpha_m, \alpha_{A'}).
```

Algorithm 2: The Attribute Analyser Algorithm.

Note that A' can be possibly empty, in which case  $q_{A'} = q_{\emptyset}$ , and the algorithm returns the answer to  $q_{\mathbf{b}} \wedge q_{\emptyset} = q_{\mathbf{b}}$  only. Likewise, **B** can be possibly empty, meaning that  $q_{\mathbf{b}} = q_{\emptyset}$ , in which case we are analysing A' over the whole dataset D (rather than over a sub-population).

**Example 2.** Consider the dataset from Table 1. Let  $\mathbf{B} = (\text{Suburb, Gender})$ . Furthermore, let  $\mathbf{b} \in \mathbf{B}$  be (Redfern, M). Thus, we are interested in the sub-population of people who are male and living in the suburb of Redfern in the dataset. Thus  $q_{\mathbf{b}} = q_{\text{Redfern}} \wedge q_{\text{M}}$ . Let A = Age, and  $A' \subseteq A$  be {20-29, 30-39}. Then  $\alpha_1$  corresponds to  $q_{\mathbf{b}} \wedge q_{20-29}$  (true answer 2),  $\alpha_2$  corresponds to  $q_{\mathbf{b}} \wedge q_{30-39}$  (true answer 0), and  $\alpha_{A'}$  corresponds to  $q_{\mathbf{b}} \wedge q_{A'} = (q_{\mathbf{b}} \wedge q_{20-29}) \vee (q_{\mathbf{b}} \wedge q_{30-39})$  (true answer 2). If on the other hand, we have B = Suburb and b = Redfern, A = Gender and  $A' = \{M, F\}$ , then we get  $(q_b \wedge q_{\text{M}})(D) = 2$ ,  $(q_b \wedge q_F)(D) = 1$ , and  $(q_b \wedge q_{A'})(D) = 3$ . Note that  $C(q_b \wedge q_{\text{M}}) \neq C(q_b \wedge q_F) \neq C(q_b \wedge q_{A'})$ . Thus,  $\mathcal{M}$  would add fresh noise values to each of these true counts. If the suppression parameter s is set to 1, then the answer to  $q_b \wedge q_F$  would be fixed to 0.

## 4 Privacy Attacks

We assume an attacker (say, a curious analyst) who is given oracle access to the Attribute Analyser (which in turn uses the Bounded Noisy Counts algorithm as a subroutine). The attacker may not know the parameters r. We describe two attacks on the algorithm. The first attack finds the hidden perturbation parameter r (if it is assumed not to be public knowledge). The second attack removes the noise to obtain the original count n = q(D) of some query of choice q. We also show how to use this attack to obtain the value  $q_a(D)$  for some target attribute value  $a \in A$ , and then for all attribute values  $a \in A$ . We remark that none of the attacks depend on any background information. For simplicity, we assume that s = r.

## 4.1 Attack 1: Finding the Perturbation Parameter r

Let  $b \in B$  be an attribute value and let  $A \neq B$  be an attribute with only two attribute values  $a_1$  and  $a_2$ . Let  $n = q_b(D)$ ,  $n_1 = (q_b \land q_{a_1})(D)$  and  $n_2 = (q_b \land q_{a_2})(D)$ . Consider the sequence of inputs  $(b, \{a_1\})$ ,  $(b, \{a_2\})$  and  $(b, \emptyset)$  to the Attribute Analyser. As output, we obtain  $n_1 + e_1$ ,  $n_2 + e_2$  and  $n + e_3$ , where  $e_i$  are the noise terms added by Bounded Noisy Counts. Clearly,  $n = n_1 + n_2$ . Furthermore,

**Lemma 1.** If  $n_1, n_2 > r$ , then  $e_1, e_2$  and  $e_3$  are independent samples from the distribution  $\mathbb{U}_{\pm r}$ .

 $<sup>^5</sup>$ This models a target sub-population in the dataset D.

Proof. Since  $n_1$  and  $n_2$  are both greater than r, the noisy answers returned by Bounded Noisy Counts are non-zero. Furthermore, since  $n_1, n_2 > r > 0$ , we see that  $\{C(q_b \wedge q_{a_1}), C(q_b \wedge q_{a_2})\}$  is a partition of  $C(q_b)$ . Hence, the two have necessarily different contributors:  $C(q_b \wedge q_{a_1}) \neq C(q_b \wedge q_{a_2})$ . Therefore, Bounded Noisy Counts adds independent noise to the corresponding queries. Furthermore,  $C(q_b) \neq C(q_b \wedge q_{a_1})$  and  $C(q_b) \neq C(q_b \wedge q_{a_2})$ , since the cardinality of both are greater than r, and hence cannot be equal to the total. Therefore, there is independent noise added to n as well.

**Lemma 2.** Let  $b_1, \ldots, b_m$  be different attribute values from one or more attributes. If  $\mathcal{M}(q_{b_i}) \neq \mathcal{M}(q_{b_j})$  then  $C(q_{b_i}) \neq C(q_{b_i})$ , for all  $i, j \in [m]$ ,  $i \neq j$ .

*Proof.* Assume the contrapositive for some i and j. Then since  $C(q_{b_i}) = C(q_{b_j})$ , the Bounded Noisy Counts algorithm should add the same noise to  $q_{b_i}$  and  $q_{b_j}$ , and hence  $\mathcal{M}(q_{b_i}) = \mathcal{M}(q_{b_j})$ ; a contradiction.

Now, define the random variable

$$Z = (n_1 + E_1) + (n_2 + E_2) - (n + E_3)$$
  
=  $E_1 + E_2 - E_3$  (1)

where  $E_i$  are i.i.d. random variables with distribution  $\mathbb{U}_{\pm r}$ . Since  $E_i \leq r$ , we have  $Z \leq 3r$ . Our attack can be summarised as follows:

- 1. Find an attribute A with only two attribute value  $a_1$  and  $a_2$  (e.g., gender).
- 2. Find m different attribute values  $b_1, \ldots, b_m$  from any number of attributes such that  $\mathcal{M}(q_{b_i} \wedge q_{a_1})$  and  $\mathcal{M}(q_{b_i} \wedge q_{a_2})$  are greater than 0 implying that  $\mathcal{M}(q_{b_i}) > 0$ . This ensures the condition of Lemma 1. Furthermore, ensure that the contributors of all queries  $q_{b_i}$  are different. See Lemma 2 to see how to ensure that.
- 3. For the *i*th attribute value, obtain  $z_i$ , an instance of the random variable Z in Eq. 1.
- 4. Let  $z_{\text{max}}$  be the maximum of the m values. We then return  $\left\lceil \frac{z_{\text{max}}}{3} \right\rceil$  as the guess for r.

We can in fact do better by also keeping track of the minimum values. Let  $z_{\min}$  be the minimum of the m values. Notice that  $Z \geq -3r$ . Our guess for r is then  $\max\{-\lceil \frac{z_{\min}}{3} \rceil, \lceil \frac{z_{\max}}{3} \rceil\}$ . The guess for r would be correct as long as either  $-z_{\min}$  or  $z_{\max}$  is greater than 3(r-1). The exact algorithm is described in Algorithm 3.

```
Input: m unique attribute values b_1, \ldots, b_m, attribute A of cardinality 2 with attributes a_1 and a_2, all satisfying \mathcal{M}(q_{b_i} \wedge q_{a_1}), \mathcal{M}(q_{b_i} \wedge q_{a_2}) > 0 and \mathcal{M}(q_{b_i}) \neq \mathcal{M}(q_{b_j}), for i, j \in [m], i \neq j.

1 Set z_{\min} \leftarrow \infty and z_{\max} \leftarrow -\infty.
2 for i = 1 to m do
3 | Run Attribute Analyzer with inputs (b_i, \{a_1\}), (b_i, \{a_2\}) and (b_i, \emptyset) and get outputs z_1, z_2 and z_3, respectively.
4 | Set z = z_1 + z_2 - z_3.
5 | if z > z_{\max} then
6 | z_{\max} \leftarrow z.
7 | if z < z_{\min} then
8 | z_{\min} \leftarrow z.
9 Let z' = \max\{-\lceil \frac{z_{\min}}{3} \rceil, \lceil \frac{z_{\max}}{3} \rceil\}.
10 Output z'.
```

**Algorithm 3:** The Perturbation Finder Algorithm.

We will show that the algorithm returns the correct perturbation r with high probability, depending on a suitable choice for m.

<sup>&</sup>lt;sup>6</sup>This would happen if  $E_1 = E_2 = r$  and  $E_3 = -r$ .

**Lemma 3.** Let  $r \ge 1$ , and let  $E_1$ ,  $E_2$  and  $E_3$  be variables that take values in  $\mathbb{Z}_{\pm r}$ . Out of the  $(2r+1)^3$  possible values of the tuple  $(E_1, E_2, E_3)$ , there are precisely 20 that satisfy  $E_1 + E_2 + E_3 > 3(r-1)$  or  $E_1 + E_2 + E_3 < -3(r-1)$ .

Proof. First let us consider the number of permutations whose sum is greater than 3(r-1). Note that none of the  $E_i$ 's can be less than r-3. To see this, note that if r=1, then any of the  $E_i$ 's cannot be equal to r-3=-2. Let us assume that r>1, then if any of the  $E_i$ 's is  $\leq r-3$ , then the maximum possible sum is  $\leq r-3+r+r=3(r-1)$ , which is our threshold. Thus, we enumerate all possible permutation of values of the  $E_i$ 's, such that  $E_i \geq r-2$  and  $E_1+E_2+E_3>3(r-1)$ . This is shown below.

$E_1$	$E_2$	$E_3$
r	r	r
r	r	r-1
r	r	r-2
r	r-1	r
r	r-1	r-1
r	r-2	r
r-1	r	r
r-1	r	r-1
r-1	r-1	r
r-2	r	r

There are exactly 10 such values. By symmetry, the same holds for  $E_1 + E_2 + E_3 < -3(r-1)$ .

**Proposition 2.** Let r' be the output of the perturbation finder. Then

$$\Pr[r' = r] = 1 - \left(1 - \frac{20}{(2r+1)^3}\right)^m$$

*Proof.* From Lemma 3, there are exactly 20 possible values of the tuple  $(z_1, z_2, z_3)$ , for which z in step 4 of the algorithm has sum greater than 3(r-1) or less than -3(r-1). Through Lemma 1 the variables  $z_i$  are i.i.d. The probability that z for the ith set of queries to the Attribute Analyzer is within the interval [-3(r-1), 3(r-1)] is given by  $1-20/(2r+1)^3$ . The result follows for all m attributes, since the ith z in step 4 is independently distributed due to Lemma 2.

For a given probability of success, larger perturbations require more queries to  $\mathcal{M}$  (through the perturbation finder algorithm). However, note that larger values of r are not desirable from a utility point of view. Figure 1 shows the number of attributes m required for a given probability of success. Note that smaller values of r, i.e.,  $\leq 5$ , which are desirable from a utility point of view need less than m = 200 for a 95% success rate.

**Remark 1.** We have assumed for simplicity that A is an attribute with exactly two attribute values. In general, the attack is applicable to any attribute with at least two attribute values. In this case, we run the Attribute Analyser with inputs corresponding to the two selected attribute values, plus the input  $(b_i, A')$ .

**Remark 2.** Again for simplicity, in the ith iteration of Algorithm 3, we run the Attribute Analyser on three different inputs. These can be replaced by a single input  $(b_i, A')$ , where  $A' = \{a_1, a_2\} \subseteq A$ . The output of Attribute Analyser will by definition return (noisy) answers to the queries  $q_b \land q_{a_1}$ ,  $q_b \land q_{a_2}$  and  $q_b \land q_{A'}$  as desired (step 5 of Algorithm 2). Thus, while this constitutes 3 queries to Bounded Noisy Counts, this is only a single query to the Attribute Analyser. The latter resembles the TableBuilder tool interface.

<sup>&</sup>lt;sup>7</sup>Note that even though the lemma applies to the sum  $z_1 + z_2 + z_3$ , it is easy to see that it also holds true for  $z_1 + z_2 - z_3$ .

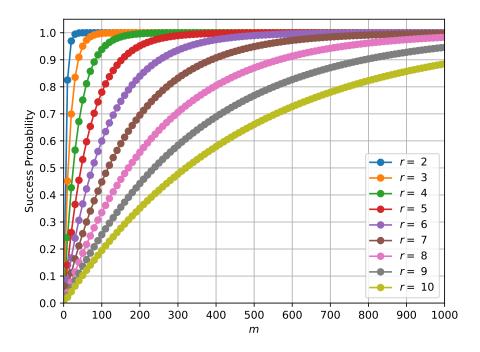


Figure 1: Probability of successfully finding the perturbation parameter r as a function of the number of attribute values m in the Perturbation Finder algorithm. Higher perturbations require a much larger number of attribute values (and hence number of queries).

### 4.2 Attack 2: Removing Noise

Consider again a tuple of attributes **B** (possibly empty) from D, and let  $\mathbf{b} \in \mathbf{B}$  denote a vector of attribute values from **B** defined as before. Consider an attribute A with m attribute values  $a_1, \ldots, a_m$ . In this section, we will show an attack that finds the exact answer to  $q_{\mathbf{b}} \wedge q_A$  by using the Attribute Analyzer as a black box. We will then show how to use this algorithm to find the true answer to any target query  $q_{\mathbf{b}} \wedge q_{a_i}$ . Continuing on, we can find the true answers to all queries  $q_{\mathbf{b}} \wedge q_{a_i}$ ,  $i \in [m]$ . We will first assume that  $C(q_{\mathbf{b}} \wedge q_{a_i}) \neq \emptyset$  for all i, for simplicity. Later on we will show that this assumption can be relaxed as long as we have some m' < m attributes from A satisfying  $C(q_{\mathbf{b}} \wedge q_{a_i}) \neq \emptyset$ . Note that Proposition 1 shows how to ensure that this condition holds.

We begin with a simple observation on A. Recall that a two-partition of a set A is a partition of A with exactly two subsets of A.

**Lemma 4.** There are exactly  $2^{m-1} - 1$  two-partitions of the set A.

*Proof.* There are  $2^m$  elements in the power set of A. Out of these, two are  $\emptyset$  and A itself. Out of the remaining  $2^m - 2$  elements (subsets of A), we can construct a two-partition by choosing any element as the first subset, say A', and A - A' as the other subset. Since A - A' is also in the power set, we have counted each two-partition twice. Thus, dividing  $2^m - 2$  by 2 gives us the result.

We will let  $P_A$  denote the set of all two-partitions of A.

**Lemma 5.** Assume  $C(q_b \wedge q_{a_i}) \neq \emptyset$ , for all  $i \in [m]$ . Let  $\mathcal{A}$  be a two-partition in  $P_A$ , and let  $A' \in \mathcal{A}$  be any of the two sets in  $\mathcal{A}$ . Let A'' be either the other set in  $\mathcal{A}$  or any of the two sets from any other partition in  $P_A$ . Then,  $C(q_b \wedge q_{A'}) \neq C(q_b \wedge q_{A''})$ .

Proof. Let  $S = A'\Delta A''$ . Then, since  $A' \neq A''$ ,  $S \neq \emptyset$ . If  $C(q_{\mathbf{b}} \wedge q_{A'}) = C(q_{\mathbf{b}} \wedge q_{A''})$ , then necessarily  $(q_{\mathbf{b}} \wedge q_a)(D) = 0$ , where  $a \in S$  (and therefore  $a \in A$ ). But this contradicts the assumption.

**Example 3.** Consider the dataset in Table 1. Let A = Suburb. Then A has m = 4 attribute values: D = Darlinghurst, N = Newtown, R = Redfern and S = Surry Hills. The  $2^{m-1} - 1 = 7$  two-partitions of A are as follows:

$A_1$	$A_2$
{D}	$\{N, R, S\}$
$\{N\}$	$\{D, R, S\}$
$\{R\}$	$\{D, N, S\}$
${S}$	$\{D, N, R\}$
$\{D, N\}$	$\{R, S\}$
$\{D, R\}$	$\{N, S\}$
$\{D, S\}$	{N, R}

Let  $\mathbf{B} = \emptyset$  and hence  $\mathbf{b} = \emptyset$ . Then  $q_{\mathbf{b}} \wedge q_{a_i} = q_{a_i}$  for all  $a_i \in A$ ,  $i \in \{1, 2, 3, 4\}$ . Also, from Table 1,  $C(q_{a_i}) \neq \emptyset$ , for all  $a_i \in A$ . Let A' be any of the 14 sets in the table above, and let  $A'' \neq A'$  be any of the remaining 13 sets. Then, according to Lemma 5,  $C(q_{A'}) \neq C(q_{A''})$ . One can easily verify through Table 1 that this is indeed true.

Let  $n = |C(q_b)| = |C(q_b \land q_A)|$ , which we seek to find through the attack. Consider a partition  $\{A_1, A_2\}$  in  $P_A$ , and note that

$$(q_{\mathbf{b}} \wedge q_{A_1})(D) + (q_{\mathbf{b}} \wedge q_{A_2})(D) = n.$$

Now consider the queries  $(\mathbf{b}, A_1)$  and  $(\mathbf{b}, A_2)$  to the Attribute Analyser. In return, among other answers, we get  $\alpha_{A_1}$  and  $\alpha_{A_2}$ , which are the noisy answers to the two (total) queries mentioned above. Adding the two, we have

$$z = n + e_1 + e_2,$$

where  $e_1$  and  $e_2$  are unknown error terms from  $\mathbb{Z}_{\pm r}$ . Our attack is as follows: for each of the  $t = 2^{m-1} - 1$  partitions in  $P_A$ , query the Attribute analyser with the two sets in the partition, add the answers, and average them over all t. The algorithm is shown in Algorithm 4.

**Input:** A vector of attribute values  $\mathbf{b} \in \mathbf{B}$ , attribute A with m different attribute values and set  $P_A$  of two-partitions of A.

- 1 Initualize  $z \leftarrow 0$ .
- **2** for each two-partition  $\{A_1, A_2\}$  in  $P_A$  do
- **3** Query the Attribute Analyzer with inputs  $(\mathbf{b}, A_1)$  and  $(\mathbf{b}, A_2)$  and obtain  $\alpha_{A_1}$  and  $\alpha_{A_2}$ .
- 4 Update  $z \leftarrow z + \alpha_{A_1} + \alpha_{A_2}$ .
- **5** Let  $t = 2^{m-1} 1$  and obtain  $z \leftarrow z/t$ .
- 6 Output  $\lfloor z \rfloor$ .

**Algorithm 4:** The Noise Remover Algorithm.

#### 4.2.1 Success Probability

Let  $Z_i$  denote the random variable denoting the sum in Step 4 of the algorithm for the *i*th partition, where  $i \in [t]$ ,  $t = 2^{m-1} - 1$ . We have

$$Z_i = n + E_1^{(i)} + E_2^{(i)}, (2)$$

where  $E_1^{(i)}$  and  $E_2^{(i)}$  are the noise variables.

**Lemma 6.** For each  $i \in [m]$ ,  $E_1^{(i)}$  and  $E_2^{(i)}$  are i.i.d. random variables with distribution  $\mathbb{U}_{\pm r}$ . Furthermore,  $Z_1, \ldots, Z_t$  as defined by Eq. 2 are i.i.d. random variables.

<sup>&</sup>lt;sup>8</sup>For any two sets A and B,  $A\Delta B$  denotes their symmetric difference.

Proof. Fix an i. The the two error variables  $E_1^{(i)}$  and  $E_2^{(i)}$  are noise terms added to the two sets of the corresponding two-partitions. The queries corresponding to the two sets by definition have different contributors (in fact, mutually exclusive). Therefore, the Bounded Noisy Counts algorithm adds independent noise with distribution  $\mathbb{U}_{\pm r}$ . Now consider,  $Z_1, \ldots, Z_t$ . By Lemma 5 the query corresponding to every set in the set of two-partitions  $P_A$  has different contributors. Hence  $E_j^{(i)}$  are i.i.d. with distribution  $\mathbb{U}_{\pm r}$ , where  $j \in \{0,1\}$ ,  $i \in [t]$ . The result follows.

Now define

$$\overline{Z} = \frac{1}{t} \sum_{i=1}^{t} Z_i.$$

The success probability of the Noise Remover is then given by

$$\Pr\left(\left|\overline{Z} - n\right| < 0.5\right)$$

Define  $Y_i = E_1^{(i)} + E_2^{(i)}$  and  $\overline{Y} = \frac{1}{t} \sum_{i=1}^t Y_i$ . Then

$$\Pr\left(\left|\overline{Z} - n\right| < 0.5\right) = \Pr\left(\left|\overline{Y} + \frac{1}{t}\sum_{i=1}^{t} n - n\right| < 0.5\right)$$
$$= \Pr\left(\left|\overline{Y}\right| < 0.5\right) \tag{3}$$

Thus, we will attempt to find  $\Pr(|\overline{Y}| < 0.5)$ . We will first show a lower bound on this probability and then an exact expression.

#### Lower Bound on the Success Probability

It is easy to see that  $\mathbb{E}(E_i^{(i)}) = 0$ . And by the variance of the discrete uniform distribution

$$Var(E_j^{(i)}) = \frac{(r - (-r) + 1)^2 - 1}{12} = \frac{r(r+1)}{3}.$$

By the linearity of expectation

$$\mathbb{E}(Y_i) = 0,$$

and by Lemma 6,

$$Var(Y_i) = \frac{2r(r+1)}{3}.$$

Again through linearity of expectation

$$\mathbb{E}(\overline{Y}) = 0,$$

and by Lemma 6,

$$Var(\overline{Y}) = \frac{1}{t^2} \sum_{i=1}^{t} \frac{2r(r+1)}{3} = \frac{2}{3} \frac{r(r+1)}{t}.$$

Using Chebyshev's inequality, we see that

$$\Pr\left(\left|\overline{Y} - \mathbb{E}(\overline{Y})\right| \ge \epsilon\right) \le \frac{\operatorname{Var}(\overline{Y})}{t\epsilon^2}$$

By setting  $\epsilon = 0.5$ , and putting in the values of  $\mathbb{E}(\overline{Y})$  and  $\text{Var}(\overline{Y})$ , we get

$$\Pr\left(\left|\overline{Y}\right| \ge 0.5\right) \le \frac{8}{3} \frac{r(r+1)}{t}.$$

Thus,

$$\Pr(|\overline{Y}| < 0.5) \ge 1 - \frac{8r(r+1)}{3(2^{m-1}-1)}.$$

Figure 2 shows lower bounds on the success probabilities against different perturbation parameters as a function of t.

**Remark 3.** Figure 2 shows that we do not need to use all the two-partitions in  $P_A$  to achieve a given probability of success. Furthermore, since each iteration calls the Attribute Analyser twice (one for each partition), we have a total of 2t calls to Attribute Analyser. Thus, if we were to run this algorithm on the TBE algorithm via the TableBuilder tool, this would require running the tool a total of 2t times.

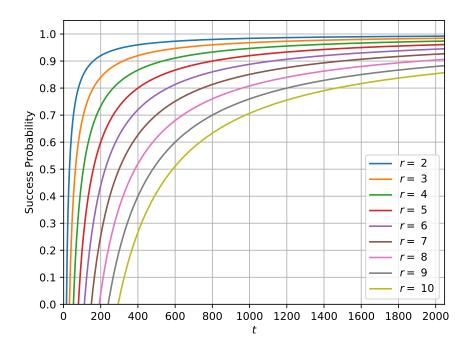


Figure 2: Lower bounds on the probability of successfully retrieving the actual count  $n=(q_{\mathbf{b}}\wedge q_A)(D)$  through the Noise Remover algorithm. Here m=12 and hence t ranges from 1 to  $2^{m-1}-1=2047$ .

## **Exact Success Probability**

Consider the sum  $\sum_{i=1}^{t} E_1^{(i)} + E_2^{(i)}$ . Simplifying notation, we can view this as the sum of 2t i.i.d. random variables  $E_i$  (due to Lemma 6). The probability mass function of each  $E_i$  is given by

$$f_E(x) = \begin{cases} \frac{1}{2r+1} & \text{if } x \in \mathbb{Z}_{\pm r} \\ 0 & \text{otherwise} \end{cases}$$
.

Let  $X_1 = E_1$ , and define  $X_i = X_{i-1} + E_i$ , for  $i \in \{2, ..., t\}$ . Then the probability mass function of  $X_2$  is given by

$$f_{X_2}(x) = \sum_{y=-\infty}^{+\infty} f_{X_1}(y) f_E(x-y) = \sum_{y=-\infty}^{+\infty} f_E(y) f_E(x-y),$$

and for every i

$$f_{X_i}(x) = \sum_{y=-\infty}^{+\infty} f_{X_{i-1}}(y) f_E(y-x).$$

Thus, we can iteratively find  $f_{X_{2t}}$ , the probability mass function of  $X_{2t}$ . Now,

$$\Pr\left(\left|\overline{Y}\right| < 0.5\right) = \Pr\left(\left|\frac{X_{2t}}{t}\right| < 0.5\right)$$

$$= \Pr\left(-\frac{1}{2} < \frac{X_{2t}}{t} < \frac{1}{2}\right)$$

$$= \Pr\left(-\frac{t}{2} < X_{2t} < \frac{t}{2}\right)$$

$$= \sum_{x \in (-t/2, t/2)} f_{X_{2t}}(x). \tag{4}$$

Thus, we can evaluate Eq. 4 to find the exact success probability of the Noise Remover algorithm to obtain the answer  $n = (q_b \land q_A)(D)$ . Figure 3 shows these success probabilities. Comparing this with Figure 2, we see that the actual success probability is higher for much smaller values of t.

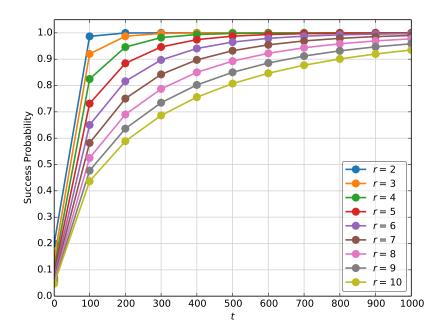


Figure 3: Probability of successfully retrieving the actual count  $n = (q_b \wedge q_A)(D)$  through the Noise Remover algorithm. Here t ranges from 1 to 1000.

#### 4.2.2 Broadening the Scope of Attack 2

We now show that our attack can be used much more broadly.

## Relaxing the Non-Empty Contributors Assumption

We assumed in the previous section that all m attribute values of A satisfy  $C(q_{\mathbf{b}} \wedge q_{a_i}) \neq \emptyset$ ,  $i \in [m]$ . From Figure 3, it is clear that we do not need all the two-partitions of A to find  $q_{\mathbf{b}} \wedge q_A$ . As long as we have an  $m' \leq m$  number of attribute values whose corresponding queries have non-empty contributors,  $q_{\mathbf{b}}$  we can use them to find the answer to the aforementioned query in the following way. Let  $a_1, \ldots, a_{m'}, a_{m'+1}, \ldots, a_m$  denote the m attribute values of A, and assume (w.l.o.g.) that only upto  $a_{m'}$  have  $\mathcal{M}(q_{\mathbf{b}} \wedge q_{a_i}) > 0$  (and

<sup>&</sup>lt;sup>9</sup>Recall that by Proposition 1 it is easy to find attribute values whose corresponding queries have non-empty contributors.

hence  $C(q_{\mathbf{b}} \wedge q_{a_i}) \neq \emptyset$ ). Therefore  $\mathcal{M}(q_{\mathbf{b}} \wedge q_{a_i})$  is 0 for all  $m' < i \leq m$ . Let A'' denote the set of attribute values  $a_{m'+1}, \ldots, a_m$ . Note that  $C(q_{\mathbf{b}} \wedge q_{A''})$  can be possibly empty. We first construct all two-partitions of the set A' = A - A'', resulting in  $2^{m'-1} - 1$  two-partitions. Denote this by  $P_{A'}$ . Then in each two-partition we add A'' to any one (but not both) of the two sets in the partition. It is easy to see that the resulting set is a set of two-partitions of A. Furthermore, we still ensure that Lemma 6 holds. For, if  $C(q_{\mathbf{b}} \wedge q_{A''})$  is empty, then Lemma 6 automatically holds due to construction of  $P_{A'}$ . On the other hand, if  $C(q_{\mathbf{b}} \wedge q_{A''})$  is not empty then the lemma follows due to Lemma 5. We shall call A' a non-empty contributors subset of the attribute A.

#### Removing the Noise on a Target Attribute Value

Let us now assume that we are interested to know the value  $q_b \wedge q_a$  for some target attribute value a in A. We take a non-empty contributors subset A' of A such that  $a \notin A'$ . We first run the Noise Remover on the set of two-partitions  $P_{A'}$  of A', obtaining count n'. We then construct  $P_{A' \cup \{a\}}$ , and run the Noise Remover algorithm again to obtain the count as n''. The answer to the above query is then n'' - n'. If  $\mathcal{M}(q_b \wedge q_a) = 0$  then we can use the trick mentioned above to construct two-partitions of  $A' \cup \{a\}$ . Let  $p_{nr}$  denote the probability of success of Noise Remover. Then, through a simple application of the union bound, the success probability is given by  $1 - 2(1 - p_{nr})$ . This requires around 2(2t) = 4t calls to the Attribute Analyser.

#### Removing the Noise on the Attribute Histogram

Continuing on with the previous example we can in fact find answers to  $q_{\mathbf{b}} \wedge q_{a_i}$  corresponding to all m attributes of A. We first construct a non-empty contributors subset A' of A (with m' number of attributes), use it once to find  $q_{\mathbf{b}} \wedge q_{A'}$ , and then add the other attribute values in A one-by-one in a manner described above to retrieve all m answers. For any target attribute value  $a' \in A'$ , we run the Noise Remover on two-partitions of  $A' - \{a'\}$ . By the union bound, the overall probability of success is given by  $1 - (m - m' + 1 + m')(1 - p_{nr}) = 1 - (m + 1)(1 - p_{nr})$ . This requires 2t(m + 1) calls to the Attribute Analyser. Figure 4 shows the success probability in finding all queries corresponding to all attribute values in some target attribute A. Here we have used t = 800, and thus |A'| has to be  $\geq 12$ . Note that this result is obtained through the union bound, and the actual success probability might be much better.

## 5 Experimental Evaluation on the TBE Algorithm

We ran Attack 2 on a synthetic dataset accessed via an API built on top of the TBE algorithm [15]. The API mimics the functionality of the TableBuilder tool from ABS. Our privacy-preserving algorithms represent an abstract mathematical model of the TBE algorithm. As such there is one significant simplification used in our mathematical model that needs specific mention. Since the TBE algorithm is meant to answer queries on-the-fly, it maintains a pre-computed table of noise (instead of freshly generated noise for queries with a new set of contributors). Since the number of queries can be much larger than the dimension of the table, a mechanism is introduced that deterministically accesses the relevant noise entry in the table. The entries in the table are themselves uniform random entries from  $\mathbb{Z}_{\pm r}$ . To ensure that same contributors receive the same noise, the contributors (users) in the dataset are assigned unique keys. When combining different contributors, the keys are XORed and then given as input to a pseudo-random number generator which in turn maps it to a perturbed value in the table [9, 12]. We see that with a big enough perturbation table, our model is a good approximation. As we shall see, the results of our attack confirms this. The aforementioned API uses the perturbation parameter r = 2. Furthermore, it returns the output "suppressed" for counts of 1 or 2. However, for the sake of our attack, we assume that the returned count is 0 (which is a more difficult problem).

We fix a target attribute in the synthetic dataset. The attribute has 107 different attribute values. We then run the Noise Remover on each attribute value with different values of t (number of two-partitions). The results are shown in Table 2 in Appendix A. As an example, with t = 200, the probability that all

 $<sup>^{10}</sup>$ Recall that t cannot be greater than  $2^{m'-1}-1$ . While m'=11 would suffice to find all attribute values in A-A', we require m'=12 so that we can find the attribute values within A' as well.

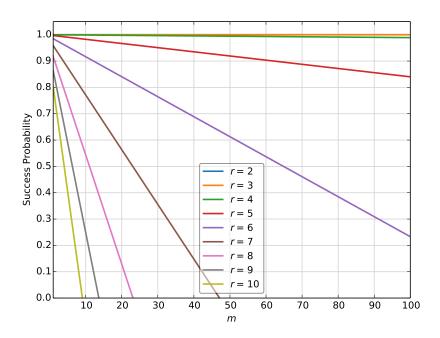


Figure 4: Probability of successfully retrieving all actual counts of the queries  $(q_{\mathbf{b}} \wedge q_{a_i})(D)$  of an attribute A with m different attribute values. Here t = 800.

107 attribute values are returned correctly is at least 0.959 (according to the analysis above, using a union bound). In practice with t=200 and t=255 all attribute values are returned exactly without any error. We see that even with t=50 (which amounts to 2t=100 queries per attribute value), we have only a 7.48% of attribute values with an incorrect answer. A few observations are in order.

- First, even for the cases where the actual count returned is incorrect, the level of noise is reduced (i.e., it is  $\pm 1$  instead of  $\pm 2$ ).
- Secondly, in some cases the noise returned is -1. By the properties of the algorithm, we can fix this to 0. This results in even less percentage of erroneous attribute values: 3.74% error for t = 50 and 0.9% (only one incorrect guess) for t = 127.
- A final observation is that the probabilities reported in Figure 4 are for the entire column. If the target is only one specific set of attribute values (corresponding to a target individual), then the probabilities are higher, i.e., 0.9996 for the case of t = 200.

## 6 Mitigation Measures

We briefly discuss possible mitigation measures against our attacks. Possible defence mechanisms against the two attacks are discussed separately.

#### 6.1 Mitigation Measures against Perturbation Finder

Recall that the attack algorithm on finding the perturbation value relies on identifying an attribute with at least two attribute values. Assume this to be the attribute gender, with attribute values male and female. The attack involves submitting queries on the number of males, the number of females, and the combined number of males and females. The last query is what we call a "total" query.

#### **Query Auditing**

An immediate defence mechanism is to audit the queries to check if an analyst is trying to find the perturbation parameter. Query auditing is in general a difficult problem [5]. Specific to finding the perturbation parameter, this defence measure needs to identify all possible query combinations that can be used to narrow down the possible range (of the perturbation parameter). The specific construction of queries (outlined above) in our attack is one possible way. But there may be other combination of queries which could be used to find the perturbation parameter. Furthermore, it is difficult to detect if there is malicious intent behind a given series of queries, as the queries can be contextually benign, e.g., an analyst might very well be checking the gender distribution across different occupations in a geographic area.

#### Query Throttling

Another alternative is to throttle the number of queries. This can be done by introducing a "cap" on the number of queries allowed to an analyst. However, in light of our results, this would be too small a number, e.g., not allowing more than 200 queries if r=5 is used as the perturbation parameter. Query throttling could also be accomplished by applying a rate limit to slow down submission of successive queries. This has an obvious adverse impact on usability.

## Eliminating the Total Query

Recall that the attack algorithm works by examining the difference between the noisy count of the total query versus the sum of noisy counts of the sub-queries. Thus one way to mitigate the attack is to not add "fresh" noise to the total query (and instead report the sum of the noisy counts from the sub-queries). Unfortunately, this significantly impacts utility. For instance, if an analyst is interested in the number of people living in a certain geographic area (say the suburb Redfern), then the only way to obtain this answer would be to add the answers obtained from the number of males and the number of females living in the area. The problem is further exacerbated by the fact that there might be multiple attributes with two attribute values under the same geographic area. And thus the attack can be (slightly) modified to instead equate the sums obtained from multiple pairs of attribute values.

#### Disclosing the Perturbation Parameter

In light of the shortcomings of the above mentioned defence measures, an inevitable choice is to make the perturbation parameter public. Apart from having a negligible impact on (individual) privacy, this is beneficial from a utility point of view as well. The analyst now knows the degree to which an answer is possibly perturbed, and can factor this amount into his/her calculations.

#### 6.2 Mitigation Measures against Noise Remover

Recall that the attack on removing noise relies on creating two-partitions of a target attribute, and the fact that fresh noise is added to the answers to the total queries from the two-partitions.

### Query Auditing

Once again, query auditing is one of the first defence mechanisms that comes to mind. Automated checks could be applied to see if a significant number of queries correspond to different two-partitions of the same sub-population. Several issues make this a less than ideal solution. First, malicious queries might not be successively submitted; a clever attacker might inject these queries in between several innocuous queries. In general, query auditing can be computationally infeasible. Secondly, we could modify the attack to include three-partitions instead of two-partitions (with a corresponding increase in the number of queries required to remove noise). Lastly, while we have demonstrated one way in which multiple answers can be combined together and averaged to remove noise, we have not checked and confirmed whether there exist other query combinations which could do the same.

<sup>&</sup>lt;sup>11</sup>In fact, the need to dispense with query auditing is one of the motivations behind the rigorous definition of differential privacy [5].

#### Query Throttling

Placing a cap on the allowed number of queries is another option, with the obvious drawback that it limits the analyst to a much smaller number of queries. The reconstruction attacks described in this paper as well as prior work on reconstruction attacks [2] suggest that this is unavoidable if bounded noise mechanisms are deemed indispensable. A technical difficulty is proposing a quantitative bound on the number of queries allowed. For instance, our results show that even 100 queries remove the noise for most attributes with a perturbation parameter of r = 2.

#### Eliminating the Total Query

The noise removing algorithm relies on the fact that answer to the total query adds fresh noise, which can be compared against the sum of the noisy counts of queries corresponding to the two-partitions. If the total query does not add fresh noise, the sum would be noisier and as a result would require a larger number of queries to eliminate noise. However, as discussed before, this is not desirable from a utility point of view. For instance, if the analyst wishes to know the number of people in a sub-population with age greater than 50, then the only way to obtain this would be to add the result obtained from each age grouping (and thus obtain a noisier answer).

#### **Provably-Private Alternatives**

Our attack is another example of a series of attacks demonstrating that high accuracy cannot be guaranteed for too many queries due to privacy concerns. For instance, prior results have shown that noise needs to be calibrated according to the number of queries to avoid database reconstruction attacks [7, 2]. Thus, a safe way of releasing noisy answers is to scale the noise as a function of the number of queries asked. Differential privacy [3, 5] is a privacy definition and framework that allows to do this. The parameter  $\epsilon$  in differential privacy determines the noise added to query answers and can be tweaked to find a balance between privacy and utility. Furthermore, this parameter can be safely disclosed without effecting privacy. However, this suggests that answering too many queries will result in noise that is not good enough for utility. This is an inherent limitation of any privacy-preserving mechanism. In particular, there is growing amount of evidence (including this work) that suggests that any meaningful guarantee of privacy cannot allow extremely accurate answers to an unlimited number of queries.

### 7 Discussion

- We reiterate that due to ethical reasons we did not demonstrate the attack on real census data, but rather showed the vulnerability of the perturbation method by applying it on a synthetic dataset. Thus, the actual TableBuilder tool is vulnerable to our attack and remains at risk from similar attacks. Notice that even though the TableBuilder tool is not equipped with an API the attack could still be performed in an automated way, e.g., one could use web-based scripts to query the tool.
- One may argue that our disclosure of the attack is potentially harmful as it instructs real-world attackers on how to attack the TableBuilder tool sitting behind Australian census data. There are a number of issues with this line of thinking. First, transparency in the design and implementation of security and privacy critical systems is of utmost importance. It allows for a thorough investigation of a system's security/privacy properties and identification of any vulnerabilities. Patching these vulnerabilities in advance mitigates the chances of inadvertent future data breaches. Secondly, it is difficult to be certain that any third party is not already aware of such attacks. Third, since the TableBuilder tool allows access to public data, it is important for the public to know the security/privacy measures in place behind usage of their data.
- We communicated the vulnerability to ABS. They acknowledged that the attack relates to Table-Builder. In response, we were told that the ABS is bringing some upcoming changes to the TableBuilder tool. These include applying user-specific cap on the number of queries (users will have to re-apply once their query quota expires), only allowing highly aggregated data to TableBuilder Guest (which

can be accessed without registration), and monitoring/auditing of TableBuilder usage logs. We believe that a more controlled access to TableBuilder is definitely a step in the right direction. For instance, only allowing access to trusted users. <sup>12</sup> As mentioned above, however, query capping/throttling and auditing are mitigation measures that are difficult to implement and impose. It is not clear exactly how many queries are safe, and as far as auditing is concerned, it is extremely difficult to determine if a series of queries is launched to carry out an attack or not (our attack or others).

- It is interesting to note that the TBE algorithm was designed to thwart averaging/differencing attacks through the "same contributors, same noise" principle. Our attack shows that averaging attacks can be carried out in more subtle ways. A somewhat different differencing attack on the TBE algorithm has been documented before, through which some information about a target individual can be inferred [1, 13]. The attack essentially relies on some background knowledge. For instance, suppose that we know that all n out of n+1 individuals in a particular group, identified by a vector of attributes  $\mathbf{b} \in \mathbf{B}$ , satisfy a particular attribute value  $a \in A$ , where  $A \notin \mathbf{B}$ . Further assume that the (n+1)th individual, the target, has the same background, i.e., takes on the values  $\mathbf{b}$ , and we would like to know if the individual also exhibits  $a \in A$ . By querying the TBE algorithm on  $\mathbf{b}$ , and then  $\mathbf{b} + a$ , we can tell if the individual does not exhibit a if the two answers returned by the TBE algorithm are different. However, notice that this attack is fundamentally different than our attack. One major difference being that our attack does not rely on any background information on other individuals.
- As we have demonstrated, our attack crucially retrieves low counts as well. Most importantly, it retrieves counts of 1 (which are suppressed by the TBE algorithm). Identifying such "uniques" amounts to re-identifying individuals. Some may argue that identifying a unique is not the same as identifying a real person in the population exhibiting those attribute values. However, maintaining this flimsy distinction between the two cases provides little solace; once uniques are identified, a little background information is enough to link them to real persons in the population [14].
- Finally, we would like to draw attention to a similar data usage scenario relating the United States (US) Census Bureau who seek to publish some aggregated form of the 2020 Census of Population and Housing [10]. The Bureau has internally investigated the applicability of database reconstruction attacks [7, 2, 4, 11] on the 2010 (aggregated) census data and has come to the conclusion that given the amount of information leaked per person, there is a "theoretical possibility" that the census data could be reconstructed. We first note that these reconstruction attacks are also applicable to noisy data (where noise is significantly less than the amount of statistics released). Secondly, our averaging attack can also be seen as a form of reconstruction attack, where the attacker can reconstruct target columns of the underlying dataset. Based on their findings, Garfinkel, Abowd and Martindale [10] conclude:

Faced with the threat of database reconstruction, statistical agencies have two choices: they can either publish dramatically less information or use some kind of noise injection. Agencies can use differential privacy to determine the minimum amount of noise necessary to add, and the most efficient way to add that noise, in order to achieve their privacy protection goals.

These recommendations for the US census data are inline with our suggestions for the Australian census data.

## 8 Conclusion

We have shown an averaging attack that retrieves actual values exhibited by an attribute (or one or more of its attribute values) in a dataset which can only be accessed via a privacy-preserving algorithm that adds bounded uniform noise to the answers. In line with previous research on reconstruction attack (see e.g., [7]), we show that if the number of allowed queries are above a given mark, the algorithm fails to provide privacy.

<sup>&</sup>lt;sup>12</sup>One may wonder if the user is trusted, why, then, use a perturbation mechanism at all? One reason provided to us is that the user may wish to publicly release information obtained from TableBuilder, e.g., a journalist. However, in such cases the information that the user wishes to publish, and only this information, can always be "sanitised" before publication.

We have demonstrated the attack on a synthetic dataset accessed via the TBE algorithm used for the ABS TableBuilder. While the TBE algorithm might be patched to resist the particular attack mentioned in this paper, we would like to stress that this may not be the only attack possible. A better alternative is to scale noise according to the number of queries allowed to minimise information leakage from a theoretical point of view [2]. This will guarantee that privacy is maintained in practice. We also restate that we have only considered one subset of queries (counting queries), and the attack may be applicable to other types of queries, e.g., range queries on continuous data.

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## A Results on the TBE Algorithm

Table 2 shows the result of running the Noise Remover algorithm on the TBE algorithm as mentioned in Section 5. The values coloured in red are erroneous results from the algorithms, i.e., where the output does not equal the actual count.

Table 2: Results of running Attack 2 to retrieve a target column of the synthetic dataset.

Att. value	Actual	Noisy	t = 50	t = 127	t = 200	t = 255
1	1	suppressed	1	1	1	1
2	3	0	3	3	3	3
3	6	6	6	6	6	6
4	12	12	12	12	12	12
5	33	34	33	33	33	33
6	114	116	114	114	114	114
7	199	197	199	199	199	199
8	372	373	372	372	372	372
9	677	678	678	677	677	677
10	1075	1074	1075	1075	1075	1075
11	2884	2883	2884	2884	2884	2884
12	4388	4389	4388	4388	4388	4388
13	6496	6495	6496	6496	6496	6496
14	9136	9137	9136	9136	9136	9136
15	12694	12692	12693	12694	12694	12694
16	16893	16892	16893	16893	16893	16893
17	21513	21515	21513	21513	21513	21513
18	26566	26564	26566	26566	26566	26566
19	31854	31853	31854	31854	31854	31854
20	36741	36739	36741	36741	36741	36741
21	40268	40267	40268	40268	40268	40268
22	43426	43427	43426	43426	43426	43426
23	44865	44867	44865	44865	44865	44865
24	44812	44813	44812	44812	44812	44812
25	43054	43053	43054	43054	43054	43054
26	35698	35696	35698	35698	35698	35698
27	31534	31532	31535	31534	31534	31534
28	26103	26105	26103	26103	26103	26103
29	20953	20955	20953	20953	20953	20953
30	12430	12431	12430	12430	12430	12430
31	8977	8975	8977	8977	8977	8977
32	6297	6296	6297	6297	6297	6297
33	2715	2713	2715	2715	2715	2715
34	1775	1774	1775	1775	1775	1775
35	1085	1084	1085	1085	1085	1085
36	614	615	614	614	614	614
37	377	378	377	377	377	377
38	196	195	196	196	196	196

39	53	52	53	53	53	53
40	24	25	24	24	24	24
41	14	15	14	13	14	14
42	3	0	3	3	3	3
43	4	0	4	4	4	4
44	1	suppressed	1	1	1	1
45	0	0	0	0	0	0
46	0	0	0	0	0	0
47	0	0	0	0	0	0
48	0	0	0	0	0	0
49	0	0	0	0	0	0
50	0	0	0	0	0	0
51	0	0	0	0	0	0
52	0	0	0	0	0	0
53	0	0	0	0	0	0
54	0	0	0	0	0	0
55	0	0	0	0	0	0
56	0	0	0	0	0	0
57	0	0	0	0	0	0
58	0	0	0	0	0	0
59	0	0	0	0	0	0
60	0	0	0	0	0	0
62	0	0	-1	0	0	0
63	0	0	0	0	0	0
64	0	0	0	0	0	0
66	0	0	0	0	0	0
67	0	0	-1	0	0	0
69	0	0	-1	0	0	0
	0	0	0	0	0	0
72	0	0	0	0	0	0
73	0	0	0	0	0	0
75	0	0	0	0	0	0
76	0	0	0	0	0	0
77	0	0	0	0	0	0
80	0	0	0	0	0	0
81	0	0	0	0	0	0
82	0	0	0	0	0	0
84	0	0	0	0	0	0
85	0	0	0	0	0	0
86	0	0	0	0	0	0
87	0	0	0	0	0	0
89	0	0	0	0	0	0
90 91	0	0	0	0	0	0
91	0	0	0	0	0	0
94	0	0	0	0	0	0
95	0	0	0	0	0	0
96	0	0	0	0	0	0
99	0	0	0	0	0	0
$\frac{99}{\text{A0}}$	0	0	0	-1	0	0
B0	53	52	53	53	53	53
	00	92	99	00	99	55

C0	1837	1838	1837	1837	1837	1837
D0	40259	40258	40259	40259	40259	40259
E0	16539	16537	16539	16539	16539	16539
F0	4283	4284	4283	4283	4283	4283
G0	93	95	93	93	93	93
H0	1	suppressed	1	1	1	1
<u>I0</u>	0	0	0	0	0	0
J0	0	0	0	0	0	0
K0	0	0	0	0	0	0
LO	0	0	0	0	0	0
M0	0	0	0	0	0	0
N0	0	0	0	0	0	0
O0	0	0	1	0	0	0
P0	0	0	0	0	0	0
$\overline{Q0}$	0	0	0	0	0	0
R0	0	0	0	0	0	0
S0	0	0	0	0	0	0
VV	0	0	-1	0	0	0
Error rate	-	-	7.48%	1.87%	0%	0%